ONS Problem Set 7

Wednesday, February 1, 2017

Problem 1: Information Theory

a) Calculate the entropy H of a binary state {0; 1} for which the probabilities are given as $p_0 = 1 - p_1$. Calculate the maximum of H; what values take p_0 and p_1 in that case?

b) You are given the alphabet {0; 10; 110; 111} and the probabilities
   i. {0.25; 0.25; 0.25; 0.25}
   ii. {8/15; 4/15; 2/15; 1/15}
   Calculate the entropy of the source for these cases and compare it to the mean word length.

Problem 2: Forward Error Correction (FEC) – Hamming Code

Bit errors in data signals can be corrected by adding a suitable overhead. Here, we use a (7,4)-Hamming code in which three overhead bits are added to every four input bits.

Assume that you have received the bit streams “0110011” and “1011010”. Check if the bit sequences have been received correctly, or otherwise correct them. Use the generator matrix

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

And the parity check matrix

$$H^T = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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